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ORIGINAL ARTICLE

USE OF CONTROL CHARTS FOR CONDUCTING ANOVA STUDY

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Abstract : In this paper, we have used the control charts for the empirical study of Analysis of Variance. The control charts have been used for studying different treatment effects and infer the study conducted by Analysis of Variance. Here we have shown that the concept of process control charts can be used for Factor Effect ANOVA study. We have constructed charts for study of different treatment effects and named these charts as Factor Effect Study (FES) Charts. These FES-Charts can be used to conduct the study inferred by the analysis of variance. Different examples have been picked up corresponding to equal number of observations for different levels of the factors and null hypothesis being rejected or accepted under ANOVA study. The modified control charts have been renamed as FES-Charts with the control limits being named as Lower Selection Limit and Upper Selection Limit. We have shown that the results inferred from ANOVA at 5% level of significance can be compared with the conclusion drawn from control chart whose A_3 value of the control limits is recomputed for 1.96-sigma selection limits. We have further shown that if 3-sigma control limits are kept intact then the conclusions of control chart can be compared with ANOVA conducted at 0.27% level of significance.

Key words: Analysis of variance, Control charts, Factor effects, Level of significance.

1. Introduction

On-line quality control procedure does inspection and control of quality during a manufacturing process. A manufacturing process must be stable and repeatable.

A stable process can be achieved by reducing the variability of the key parameters affecting the process. In on-line inspection and control of variability, control charts are used "for eliminating causes of variability which need not be left to chance, making possible more uniform quality and thereby effecting certain economies" [Shewhart (1930)]. According to Shewhart the controlled phenomenon is one for which it is possible to predict within limits, how its future is expected to vary based on the past experience. On the other hand, Fisher (1925) developed the analysis of variance technique that consists of separating the total variation of data-set into components due to different sources of variation. Each of these estimates of the variations is compared with the estimate of the variation due to chance factor and identified whether the variation due to the assignable cause is significant or not.

The analysis of variance procedure is used in testing of hypothesis. The hypothesis testing is a procedure that confirms or rejects a null hypothesis in favor of an alternative hypothesis, with certain level of probability of error being involved.

Some authors write that control charting and hypothesis testing are similar while other authors emphasize the differences between them. Differences of opinion are there in all areas of statistical sciences and so it is also there in the area of statistical quality control. Woodall (2000) has discussed some of the controversial issues in statistical process control and has tried to offer "a middle ground for the resolution of conflicts" wherever possible. Similar area of conflict is related to the relationship between control charting and hypothesis testing.

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1.1 Control Charts

Purpose of control charts is to check process stability by distinguishing common cause of variation from assignable causes of variation. A process is said to be stable or in statistical control if the probability distribution of the quality characteristic or the statistical model of the quality characteristic under study is constant over time. A control chart is a graph that indicates how a process changes over time. It consists of a central average line, an upper control limit and a lower control limit. The data points are plotted on the graph and the conclusions are drawn about the process variation being in control or out of control which may be caused by special causes of variation. The process is said to be in control if the statistic falls within the control limits, otherwise the process is said to be out of control. Also, non-random patterns on the chart signal an out of control situation. Identification of out of control process supports in making adjustments, improving and stabilizing the process.

1.2 Testing of Hypothesis

The origin of testing of hypothesis belongs way back to the year 1279, when the Royal Mint (London) conducted trials to test the standard of the coins produced by the mint [Curran-Everett (2009)]. The test, to assess whether the weight and composition of the coins was within the prescribed tolerances, was known as the Trial of the Pyx. The null and the alternative hypotheses were

H_0: The coins are within the prescribed tolerances.

H_1: The coins are outside the prescribed tolerances.

Neyman and Pearson (1928) presented a model of hypothesis testing that involved taking a decision based on competing null and alternative hypotheses. Moreover, the decision was associated with the cost involved in committing the two types of errors in the process of testing of hypothesis. Thus, in testing of hypothesis, the null hypothesis is accepted or rejected in relation to an alternative hypothesis, based on the statistic being lying in the acceptance region or the rejection region, with certain level of probability of error being considered. The analysis procedure used in hypothesis testing is called Analysis of Variance.

1.3 Comparison between Analysis of Variance and Control Charts

Prior to 1800 Gauss contributed to the study of sum of squares. First application of Analysis of Variance by Fisher was published in 1921 and became popular after Fisher included it in his book “Statistical Methods for Research Workers” in 1925. Analysis of variance is defined as the separation of variance ascribable to one group of causes from the variance ascribable to the other group and then comparing the variation due to different sources with that of the variation due to chance factor to decide whether variation due to the assignable cause is significant or not.

As pointed by Woodall and Faltin (1996), there is an on-going debate on the similarity between the structure of Control Charts and that of the Testing of Hypothesis. In a basic control chart, the process is said to be in control if the plotted statistic falls within the control limits and is said to be out of control otherwise. This analysis is similar to testing of hypothesis where-in if the statistic falls within the acceptance region the null hypothesis is accepted and if it falls in the rejection region the null hypothesis is rejected. The views in favor of similarity of control charts and testing of hypothesis are held by Box and Kramer (1992), Juran (1997) and Vining (1998).

According to Box and Kramer (1992), “process monitoring resembles a system of continuous statistical hypothesis testing”. Juran (1997) mentioned that control chart is a “perpetual test of significance”. Also, Vining (1998) stated that literature tends “to view the control chart as a sequence of hypothesis tests”. On the other hand according to Deming (1986), “Rules for detection of special causes and for action on them are not tests of a hypothesis that a system is in stable state”. Hoerl and Palm (1992), Wheeler (1995) and Nelson (1999) also emphasized the difference between control charting and hypothesis testing.

In this debate, Shewhart (1939) held a middle ground. He stated that “As a background for the development of the operation of statistical control, the formal mathematical theory of testing a statistical hypothesis is of outstanding importance, but it would seem that we must continually keep in mind the fundamental difference between the formal theory of testing a statistical hypothesis and the empirical theory of testing of hypotheses employed in the operation of statistical control. In the latter, one must also test the hypothesis that the sample of data was obtained under conditions that may be considered random”. Woodall and Faltin (1996) also conveyed their view-point in the favor of similarity of control charting and hypothesis testing. According to Woodall (2000), the relationship between control charting and hypothesis testing is not apparent because of the difference between Phase I and Phase II statistical quality control.

Control chart can be divided into two phases, namely, Phase I and Phase II. During phase I control charts are needed to bring the process in a state of statistical quality control. In phase I, historical data is used to construct trial control limits "to determine whether the process has been in control over the period of time where the data were collected and to see if reliable control limits can be established to monitor future production" [Montgomery (2005)]. Since in phase I, the process is reasonably stabilized, that is, the statistical distribution of the quality characteristic under study is known, in phase II, the emphasis is on process monitoring. The phase II procedure of control chart analysis assumes that the form of the distribution, along with the parameters for the quality characteristic under study, is known. Thus, if an assignable cause is present, there would be a shift in the parameter of the distribution. This analysis procedure resembles repeated hypothesis testing. In phase I, in the absence of such assumptions the control chart resembles a tool of exploratory data analysis.

Further, Woodall (2000) states that the view control charting is equivalent to hypothesis testing, is at best an over-simplification and "at worst the view can prevent the application of control charts in the initial part of phase I". So, he tries to propose a middle ground for the relationship.

2. Study Methodology

In this section, we have discussed the alternative concept of control charts for the empirical study of Analysis of Variance, while respecting the on-going debate on the similarity between the structure of Control Charts and that of the Testing of Hypothesis [Woodall (2000)]. Here, we have shown that the concept of process control charts can be used for Factor Effect ANOVA study. We have constructed charts for study of different treatment effects and named these charts as Factor Effect Study Charts. These FES-Charts can be used to conduct the study inferred by the analysis of variance. Different examples have been picked up corresponding to equal number of observations for different levels of the factors and null hypothesis being rejected or accepted under ANOVA study. Example 4.1 involves equal number of observations for different levels of a single factor and here, the null hypothesis, regarding the equality of the different levels of the factor, gets rejected under ANOVA study. Example 5.1 also has equal number of observations for different levels of a single factor, but the null hypothesis of the equality of different levels of a factor is accepted.

Before discussing the above mentioned two problems, we consider a model example of plotting control chart whose concept would be applied to construct a Factor Effect Study Chart. Consider the following example, which requires study of manufacturing process, using control charts.

3. Model Example

"The data in Table 1 gives the measurements of the axels of bicyCLe wheels. 12 samples were taken so that each sample contains the measurement of four axels. The measurements, which were more than 5 inches are given here." Trial control limits for $\bar{x}$ charts need to be obtained and it is required to comment whether the process is under control or not [Gupta (2004)].

According to the theory of control charts, the control limits and the centre line are given by [Montgomery (2005)], LCL (Lower Control Limit) = $\bar{\bar{x}} - A_3\bar{s}$, CL(Centre Line) = $\bar{\bar{x}}$ and UCL (Upper Control Limit) = $\bar{\bar{x}} + A_3\bar{s}$. Here, the sample size $n = 4$, so $A_3 = 1.628$ and from Table 1, $\bar{\bar{x}} = 142.125$ and $\bar{s} = 1.91$. So the values of the control limits are, LCL = 139.02, CL = 142.125 and UCL = 145.23.

A control chart is constructed using grouped scatter plot in SPSS 16.0 (2006). This constructed control chart is shown in Fig. 1.

From Fig. 1, it can be observed that the process is not in control as the sample means corresponding to sample numbers 5, 6, 7 and 8 lie outside the control limits. The $\bar{x}$ -chart indicates the lack of control in process mean. Now, suppose the same chart is plotted considering all the sample observations instead of only sample means. The plot is shown in Fig. 2.

From Fig. 2, not only the out of control process is CLear, but also the variability in the data-set can be observed. If the observations lying on the control line are considered to be out of control, it can be seen from

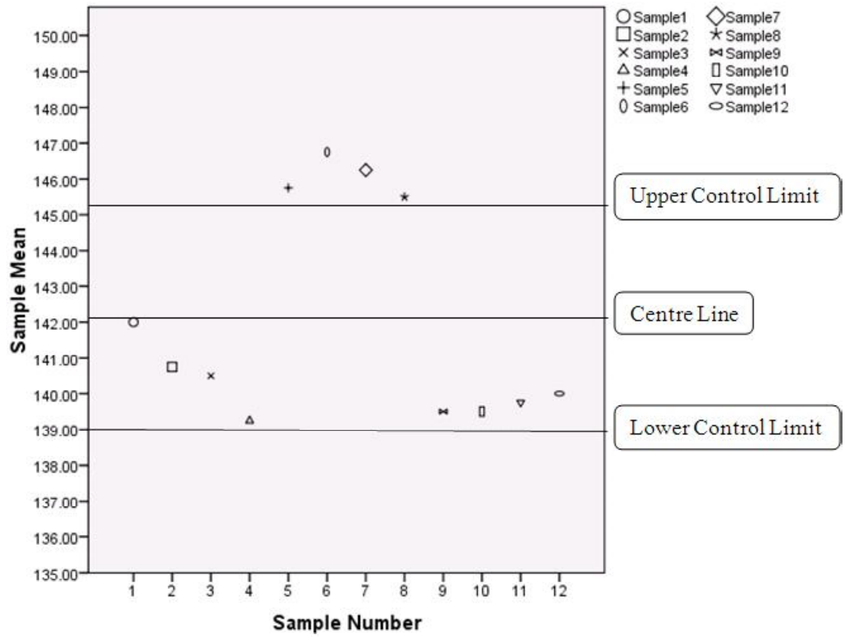

Fig. 1 : x-bar Control Chart for Measurements of Axels of Bicycle Wheels

Fig. 2 that only 50% of the observations are lying inside the control limits. Thus, obviously, the process is out of control and there is variability in the data-set. This variability is significant or not can be studied through ANOVA.

Now, suppose, instead of sample number, the observations corresponding to a factor level are considered and plotted on such a chart. This new chart is not a control chart but is modeled on the concept of a control chart. The sample numbers in the control chart taken on the x-axis are replaced by the factor levels and the observations of the samples in a control chart are replaced by the corresponding observations of each factor levels. We call this new chart as Factor Effect Study Chart.

4. Construction of Factor Effect Study Chart for Study of Inference obtained by Analysis of Variance

Example 4.1 : [Montgomery (1976), pp. 39-41] "The tensile strength of synthetic fibre used to make Cloth for men's shirts is of interest to the manufacturer. It is suspected that strength is affected by the percentage of cotton in the fibre. Five levels of cotton percentage are of interest, 15 percent, 20 percent, 25 percent, 30 percent, and 35 percent. Five observations are to be taken at each level of cotton percentage, and the 25 total observations are to be run in random order. The data obtained is shown in the Table 2.

For the given example, we would like to show that how the alternative concept of control chart can be used to study the effect of the percentage of cotton on the tensile strength of the synthetic fiber. The available technique to test "whether there is difference among the different levels of the percentage of cotton" is analysis of variance. Let us now see, how the concept of control charts can be utilized to complement the analysis of variance technique. Here, we would be replacing the sample numbers taken on the x-axis in control chart by the levels of the factor percentage of cotton.

The one-way Classification model for the data in Table 2 is $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ (1)

where, μ is general mean effect, α_i is the effect of the ith level of factor A, ε_{ij} are random variables assumed to be normally and independently distributed with mean zero and variance σ^2. $i = 1, 2, \ldots, a$, $j = 1, 2, \ldots, r$, where r is the number of repetitions.

The null and the alternative hypotheses are

$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = ... = \alpha_a = 0$ and $H_1 : \alpha_i \neq 0$ for at least one i

For the given example, a = 5 and r = 5. Analysis of variance of the data given in Table 2 is conducted and shown in the Table 3.

Here, treatment effect is significant. Thus, the null hypothesis H_0 regarding the effect of percentage of cotton on the tensile strength of synthetic fibre gets

Table 1 : Measurements (in millimeters) of axels of Bicycle wheels

	139	140	142	136	145	146	148	145	140	140	141	138
	140	142	136	137	146	148	145	146	139	140	137	140
	145	142	143	142	146	149	146	147	141	139	142	144
	144	139	141	142	146	144	146	144	138	139	139	138
$\bar{x}_i$	142	140.75	140.50	139.25	145.75	146.75	146.25	145.50	139.50	139.50	139.75	140
s_i^2	8.67	2.25	9.67	10.25	0.25	4.92	1.58	1.67	1.67	0.33	4.92	8

Table 2 : Tensile Strength of Synthetic Fibre (lb/in^2).

	Percentage of Cotton				
	15	**20**	**25**	**30**	**35**
	7	12	14	19	7
	7	17	18	25	10
	15	12	18	22	11
	11	18	19	19	15
	9	18	19	23	11
$\bar{y}_i$	9.8	15.4	17.6	21.6	10.8

Table 3 : Analysis of Variance for the Tensile Strength Data.

Source of Variation	**Sum of Squares**	**Degrees of Freedom**	**Mean Sum of Squares**	**Variance Ratio (F_0)**
Treatments	475.76	4	118.94	14.76
Error	161.20	20	8.06	
Total	636.96	24		

rejected. Hence, the percentage of cotton in the fibre significantly affects the strength.

Now, we draw a model chart using the alternative concept of control chart for the same data-set and try to analyze it. In practice, the control limits and the centre line are given by

$$\text{LCL(Lower Control Limit)} = \bar{\bar{y}} - A_3\bar{s} \tag{2}$$

$$\text{CL}(\textit{Centre Line}) = \bar{\bar{y}} \tag{3}$$

$$\text{UCL(Upper Control Limit)} = \bar{\bar{y}} + A_3\bar{s} \tag{4}$$

In the given problem, if we take r = 5 *i.e.*, the sample size corresponding to each treatment level, then A_3 = 1.427. We now plot treatment levels on the x-axis instead of samples, taken at regular intervals of time, as done for constructing control charts. From Table 2, $\bar{\bar{y}}$ = 15.04 and $\bar{s}$ = 2.8044, so LCL = 11.0581, CL = 15.04, UCL = 19.0419.

On plotting the data of Table 2 as a grouped scatter-plot, the Fig. 3 is obtained.

ANOVA in Table 3 has already examined that there is difference among the different levels of the percentage of cotton used to make the Cloth. The same can be viewed from the above chart. From the Fig. 3, it is observed that the 30% cotton produces synthetic fiber with higher tensile strength than do other treatment levels. 15% and 35% cotton results in lower mean tensile strength and among each other they do not differ much. Similarly, 20% and 25% cotton produce fibre of moderate tensile strength and have not much difference among them. Thus, the Fig. 3 also rejects the null hypothesis that the treatment effects are similar. To further probe the study and to try to identify the optimal treatment level, we can convert this scatter plot into a Factor Effect Study Chart by including the centre line and the control limits. As Factor Effect Study Chart is not a control chart, the limits drawn on FES-Chart are also not control limits, but we call them Selection Limits. The FES-Chart is obtained when upper selection limit, centre line and lower selection limit are plotted on the grouped scatter-plot as shown in Fig. 4.

As soon as the selection limits and central line are added to the chart, more Clarity in the analysis arises. If the requirement is of average tensile strength then 25% of cotton seems to be the obvious choice as for this treatment level, observations are near about mean and the variability is also least. But if the desired level of tensile strength is as high as possible then the treatment level 30% of cotton seems to be an optimal choice.

The use of the alternate concept of control chart, for the study of effects of the different levels of the factors, has enhanced the procedure of analysis of variance. Moreover, since control limits are natural

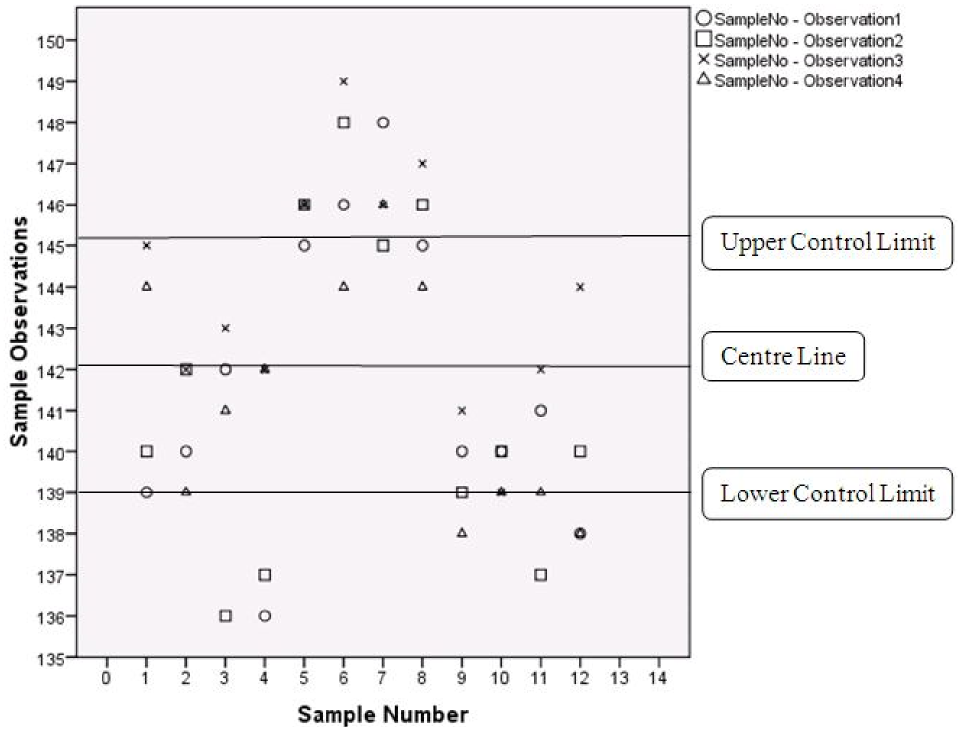

Fig. 2 : Grouped scatter plot (along with control limits) of measurements of axels of Bicycle wheels

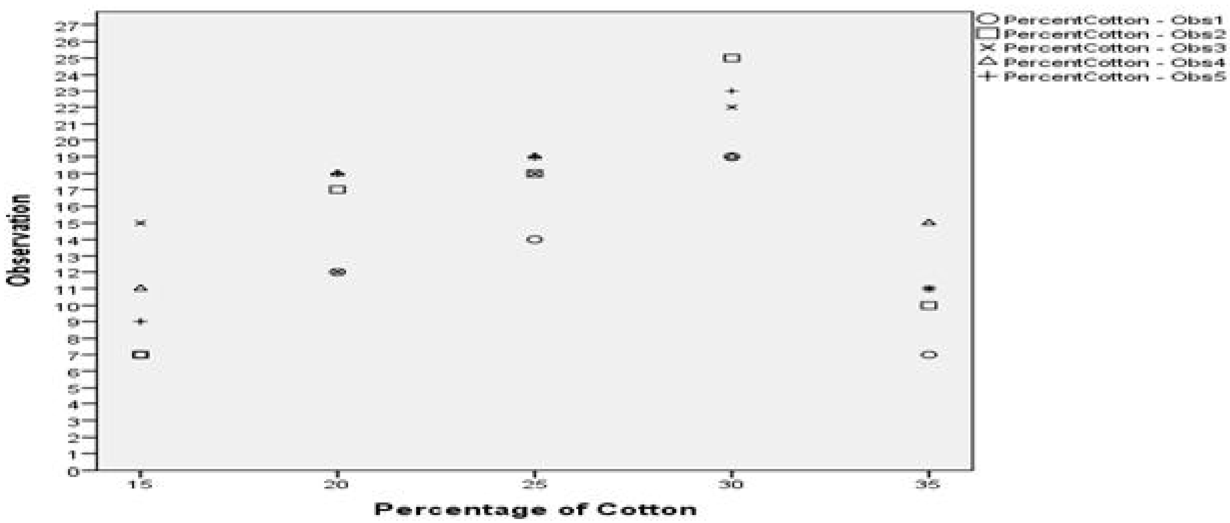

Fig. 3 : Grouped scatter plot of Tensile strength of sythetic fiber against percentage of cotton used

tolerance limits, $\hat{\mu} \pm 3\hat{\sigma}$, the level of significance is 0.27% as can be seen in the Fig. 5. If the testing is to be done at some other level of significance then the control limits will vary accordingly and the conclusion drawn may change.

For the control charts, A_3 value is determined for the 3σ control limits, for which 99.73% of the area lies in the acceptance region and 0.27% is the level of significance as shown in the Fig. 5.

But when we are conducting ANOVA at 5% level of significance, the acceptance region area is 95% as viewed in Fig. 6. So to compare the results of ANOVA and Factor Effect Study Chart either the level of significance of ANOVA needs to be kept at 0.27% so that it matches with 3σ selection limits or the A_3 value of the selection limits of the FES-Chart needs to be recomputed for 1.96σ selection limits instead of 3σ limits, so that it can be compared with ANOVA with 5% level of significance.

If ANOVA is conducted at 0.27% level of significance, we can consider the p-value of the F-Statistic to make the inference. The p-value for degrees of freedom (4, 20) of the statistic value 14.76 is

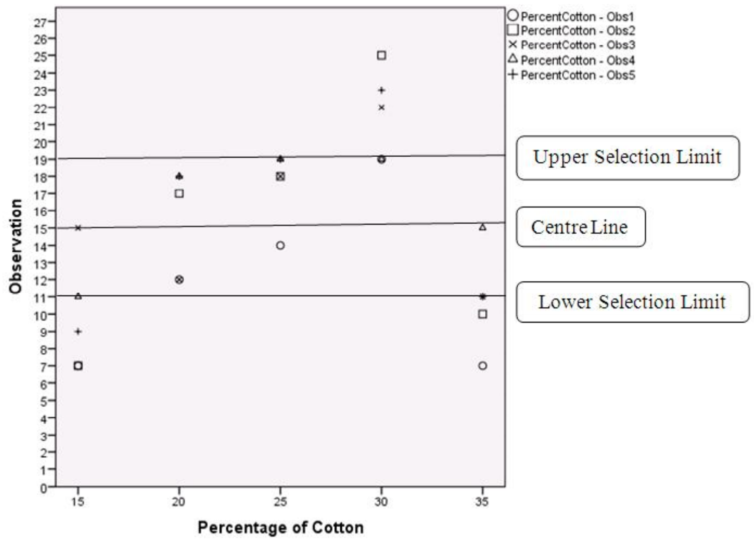

Fig. 4 : 3-σ FES-Chart for tensile strength data

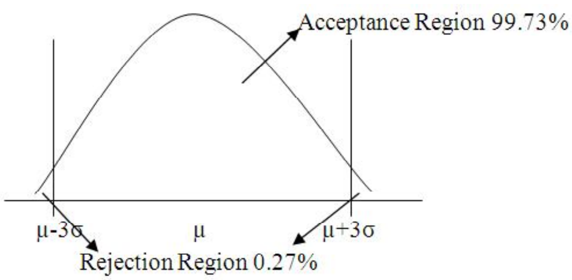

Fig. 5 : Area under the normal curve at 0.27% level of significance

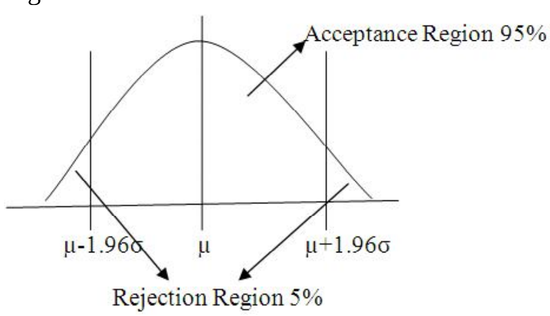

Fig. 6 : Area under the normal curve at 5% level of significance

Table 4 : Daily output on 5 randomly selected days for 4 machines

Machine I	Machine II	Machine III	Machine IV
72	62	68	64
56	70	72	72
68	66	74	68
65	64	70	68
60	78	66	58

0.00000911 [Calculator: P value for an F-test (2017)]. Thus, the null hypothesis corresponding to treatment effects of different percentage levels of cotton gets rejected at 0.27% and so also at 5% level of significance. Similarly, if instead of 3σ selection limits chart, 1.96σ selection limits chart is developed, then the $A_3 = \dfrac{1.96}{c_4\sqrt{n}}$ value would be re-computed as 0.9325, where n = 5 and c_4 = 0.94. The selection limits and the central line in this case would be LSL = 12.4249, CL = 15.04 and USL = 17.6551. With the tighter control limits, the Factor Effect Study Chart looks as shown in Fig. 7.

Viewing Fig. 7, the hypothesis corresponding to equality of treatment effects gets rejected at 5% level of significance, which is in accordance to the result of ANOVA Table 3. Also, looking at the Fig. 4, we observe that only 40% (10 out of 25) of the observations are lying inside the control limits. Thus, the hypothesis related to treatment effects needs to be rejected at 0.27% level of significance as well. This result again conforms to the ANOVA result of Table 3.

The Fig. 7 can also be interpreted for values below the lower limit and above the upper limit. The treatment effect, for which most of the values are above upper limit, produces high tensile strength cotton. The treatment effect, for which most of the values are

Table 5 : Analysis of Variance for the Machine Output Rate.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	Variance Ratio (F_0)
Treatments	94.15	3	31.38	1.0619
Error	472.8	16	29.55	
Total	566.95	19		

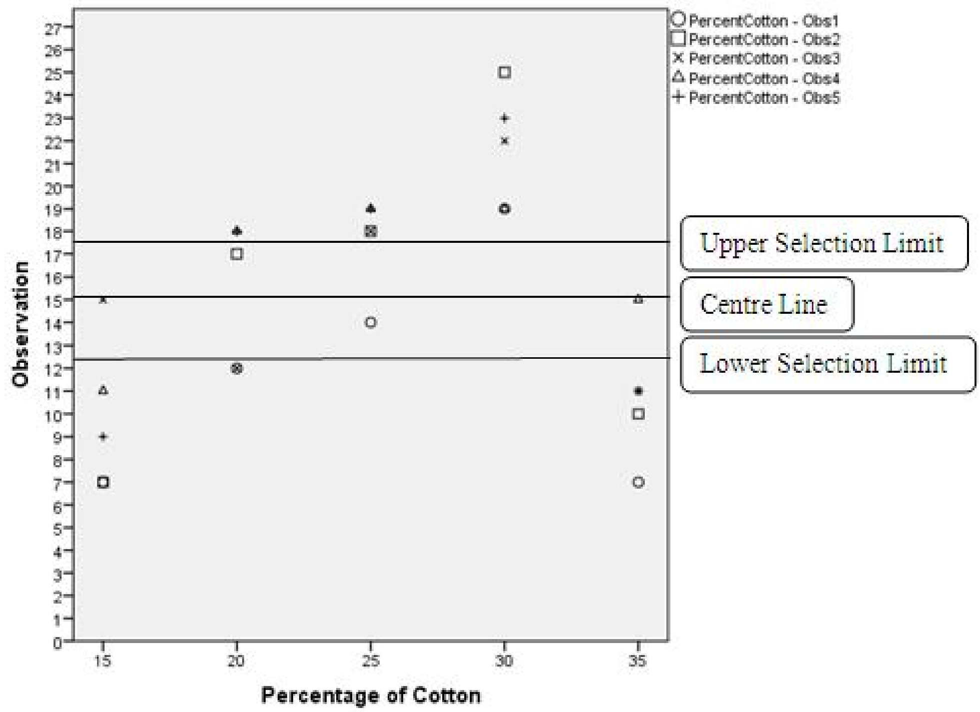

Fig. 7 : 1.96 σ FES-Chart for tensile strength data

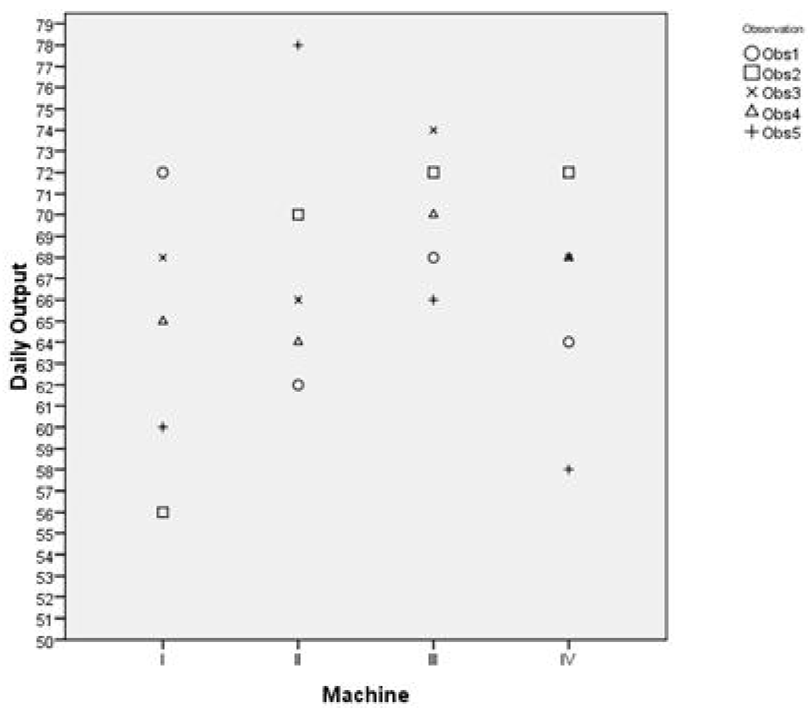

Fig. 8 : Grouped scatter plot of daily output of four different machines

below the lower limit, results in low tensile strength cotton and the treatment effect for which most of the values are near the central line indicates moderate tensile strength cotton. We have seen here that the Factor Effect Study Chart can be used for studying different treatment effects. The FES-Chart analysis is comparable to ANOVA of a design of experiment.

Let us now study the two techniques, namely the FES-Chart analysis and the ANOVA analysis for an example where the null hypothesis regarding the equality of treatment effects is accepted.

5. FES-Chart Study for Hypothesis, not Rejected

Example 5.1: [Gupta (2004), p. 23.18] "A

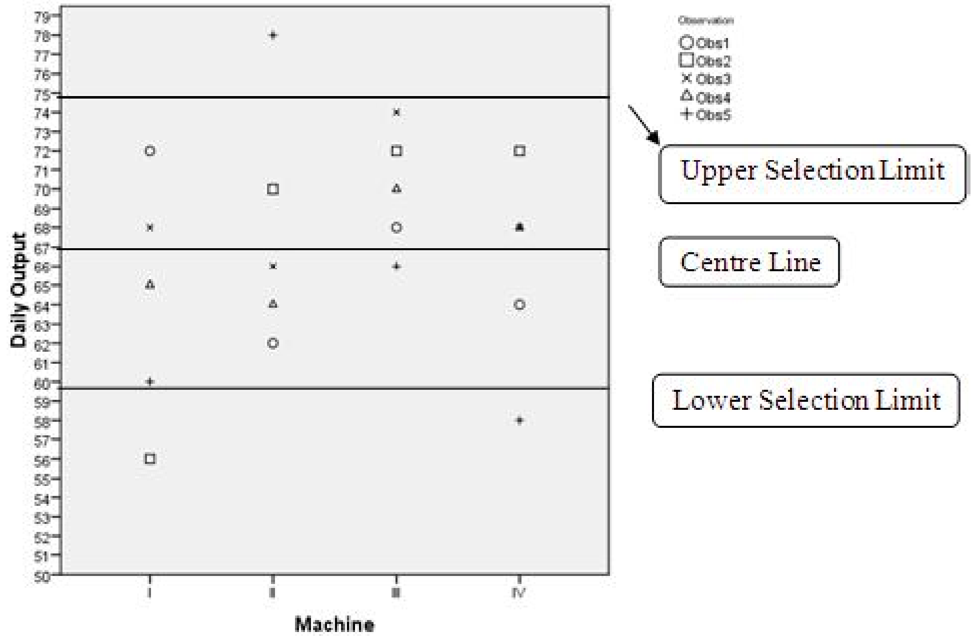

Fig. 9 : 3 σ FES-Chart for machine output data

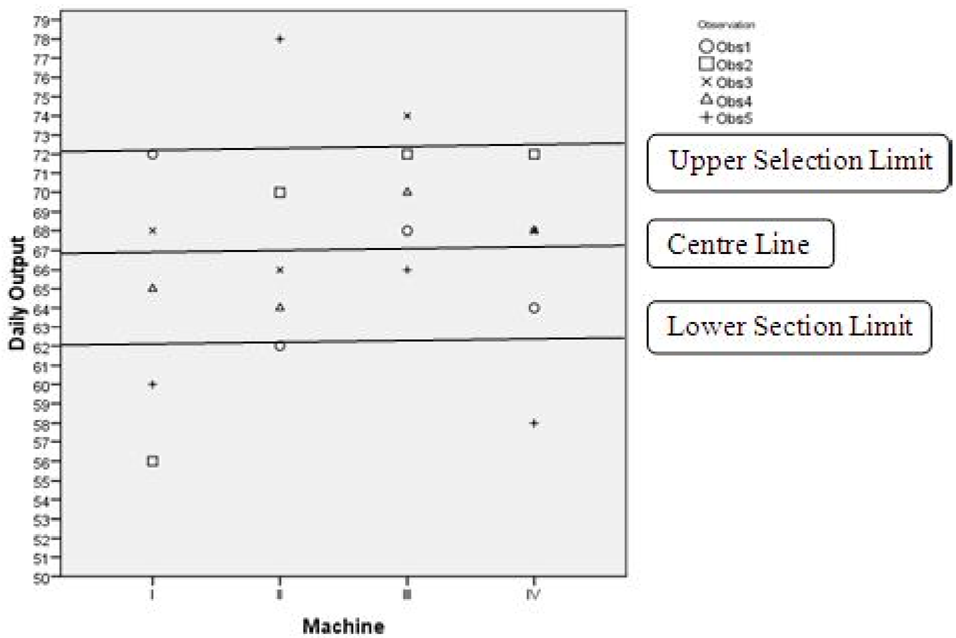

Fig. 10 : 1.96σ FES-Chart for machine output data

manufacturer of machine parts is considering one of the four machines currently in the market. The following is the daily output on 5 randomly selected days for each machine (Table 4).

Do the four machines have equal output rate?"

To test whether the four machines have equal output rate, the analysis of variance is performed in the Table 5.

Looking at the variance ratio F_0, we find that there is not much evidence to reject H_0, the hypothesis related to equality of the effects of the four machines on the output rate. The p-value for the degrees of freedom (3, 16) of the F-statistic value 1.0619 is 0.39280617 which suggests that the hypothesis would be accepted at 5% level of significance and so obviously at 0.27% level of significance. Thus, treatment effects seem to be similar for all four different machines. We would like to verify the inference by drawing the Factor Effect Study Chart for the same data. First, we consider the grouped scatter-plot for the data without including the centre line and the selection limits. Later on, we would add these lines to the scatter plot and try to infer the plot more minutely. The grouped scatter-plot of the data given in Table 4 is shown in Fig. 8.

Observing the graph, we find that daily output on different machines do not differ widely. The same observation is made in the ANOVA study, done in Table 5. Further details can be identified by converting the grouped scatter-plot to Factor Effect Study Chart by including the selection limits and centre line in the chart.

For this example,

$$\bar{\bar{y}} = 67.05, \bar{s} = 5.2797, A_3 = 1.427$$

The values of the 3σ selection limits and the centre line are

LSL = 59.52, CL = 67.05, USL = 74.58

The grouped scatter-plot, after including the lower selection limit, centre line and upper selection limit, gives the 3σ FES-Chart. This 3σ FES-Chart is shown in Fig. 9.

As 85% (17 out of 20) of the observations fall within the limits, acceptance of H_0 is justified at 0.27% level of significance. To compare the chart results with the ANOVA inference at 5% level of significance, the 1.96 σ selection limits are constructed. Now A_3 is taken as 0.9325, so that LSL = 62.13, CL = 67.05, USL = 71.97. The 1.96 σ FES-Chart is shown in Fig. 10.

Even with the tighter selection limits, it is viewed in the Fig. 10, that the effects of the different machines are not significant on the daily output of the machine parts, as 70% (14 out of 20) of the observations lie within the lower selection and upper selection limits. This is comparable with the ANOVA inference conducted at 5% level of significance in the Table 5. Further study of the FES-Chart suggests that the IIIrd machine produces output with least variability. Even if it is required to meet the target of maximum output per day, machine III seems to be more favorable to machine II because of lesser variability.

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